Lines $L_1$ and $L_2$ are given parametrically with $L_1$ the set of all points whose position vectors are
$$t(1, 1, 1) + (0, 1, 1)$$
and $L_2$ those with position vector
$$t(2, 0, 1) + (1, 1, 0).$$
Find the distance between the lines $L_1$ and $L_2$.

**Hint:** Find parallel planes $P_1$ and $P_2$ such that $P_1$ contains $L_1$ and $P_2$ contains $L_2$. Calculate the distance between $P_1$ and $P_2$.

**Solution:** $P_1$ and $P_2$ must have a common normal vector which is perpendicular to direction vectors of each of the lines. Thus we can take the normal

$$\mathbf{n} = (1, 1, 1) \times (2, 0, 1) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{vmatrix} = \mathbf{i} + \mathbf{j} - 2\mathbf{k} = (1, 1, -2).$$

The planes can be described as follows $P_1$ is
$$x + y - 2z = -1$$
and $P_2$ is
$$x + y - 2z = 2.$$

We have seen that the distance between these two planes is given by
$$\frac{|\mathbf{n} \cdot (\mathbf{a} - \mathbf{b})|}{|\mathbf{n}|}$$
with $\mathbf{a}$ a position vector of a point in $P_1$ and $\mathbf{b}$ a position vector of a point in $P_2$.

Here we can take $\mathbf{a} = (0, 1, 1)$ and $\mathbf{b} = (1, 1, 0)$. Now $|\mathbf{n}| = \sqrt{1 + 1 + 4} = \sqrt{6}$ and

$$|\mathbf{n} \cdot (\mathbf{a} - \mathbf{b})| = |(1, 1, -2) \cdot (-1, 0, 1)| = |-1 + 0 - 2| = 3.$$

So the answer is $\frac{3}{\sqrt{6}} = \frac{\sqrt{6}}{2}$. 

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