1. The matrix is \([v_1 \ v_2 \ v_3]\).

2. (a) Set \(L(x) = 0\) and solve for \(x\).
   For A, \(x_2 = x_1\) and \(x_3 = -x_1\). So \(\ker(L)\) is spanned by \((1, 1, -1)^T\).
   For B, \(x_3 = x_1\) and \(x_2 = -x_1\). So \(\ker(L)\) is spanned by \((1, -1, 1)^T\).

   (b) There are an infinite number of possibilities. The easiest choice (since it takes
   little thought and little calculation) is probably \(L(i), L(j)\) and \(L(k)\) since \(i, j\) and
   \(k\). You can compute the actual values.

   (c) \(A: \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \quad B: \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{pmatrix}\)

   (d) Since the dimension of the range plus the dimension of the kernel equals the
   dimension of the whole space, we have \((\text{ans}) + 1 = 3\) and so the answer is 2.

3. We are given that \(B = S^{-1}AS\). Taking inverses, we have
\[
B^{-1} = (S^{-1}AS)^{-1} = S^{-1}A^{-1}(S^{-1})^{-1} = S^{-1}A^{-1}S.
\]

4. This is the same problem on both exams, but the names have been changed. Suppose
   \(X\) is a subspace of \(Y\) is a subspace of \(\mathbb{R}^n\). We must prove \(Y^\perp\) is contained in \(X^\perp\).

   Suppose that \(v \in Y^\perp\). This means that \(v^T w = 0\) for every \(w \in Y\). Since \(X\) is
   contained in \(Y\), it follows that \(v^T w = 0\) for every \(w \in X\). Thus \(v \in X^\perp\).