1. (6 pts) Find a matrix $T$ so that, if $x \in \mathbb{R}^3$ has coordinates $c$ in the basis

$$
\begin{align*}
\mathbf{v}_1 &= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, & \mathbf{v}_2 &= \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}, & \mathbf{v}_3 &= \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix},
\end{align*}$$

then it has coordinates $Tc$ in the standard basis $\mathbf{i}, \mathbf{j}, \mathbf{k}$ for $\mathbb{R}^3$.

2. (24 pts) A linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ is given by $L(x) = \begin{pmatrix} x_1 + x_2 \\ x_2 + x_3 \\ x_1 - x_3 \end{pmatrix}$.

(a) What is the kernel of $L$?
(b) Find a set of vectors that span the range of $L$. (They need not be a basis.)
(c) Find a matrix $A$ such that $L(x) = Ax$.
(d) What is the dimension of the range of $L$? Give a reason for your answer.

3. (10 pts) Suppose that $A, B \in \mathbb{R}^{n \times n}$ are nonsingular and that $A$ and $B$ are similar. Prove that $A^{-1}$ and $B^{-1}$ are similar.

4. (10 pts) Suppose $V$ is a subspace of $\mathbb{R}^n$ and $W$ is a subspace of $V$. Prove that $W^\perp$ contains $V^\perp$.

**WARNING:** The final exam will probably not be in this room.