1. (18 points) Fill in the blanks with either functions, numbers or words. You need not copy the statements — just write what goes in the blanks. Some examples of words you might or might not use are

harmonic irrotational potential scalar solenoidal vector
curl divergence gradient laplacian

(a) If \( \nabla \times \mathbf{F} = \nabla \times \mathbf{G} \) for all \( \mathbf{R} \), then \( \mathbf{F} - \mathbf{G} \) is the _____ of some _____ function.

(b) If \( \nabla^2 f(\mathbf{R}) = \nabla^2 g(\mathbf{R}) \) for \( |\mathbf{R}| \leq 1 \) and \( f = g \) for \( |\mathbf{R}| = 1 \), then
\[
 f(\mathbf{R}) - g(\mathbf{R}) = _____ \text{ for } |\mathbf{R}| \leq 1.
\]

(c) If \( \nabla^2 f = 0 \), we call \( f \) a (or an) _____ function.

(d) The fundamental theorem of vector analysis states that a nice vector function in a nice domain can be written as the sum of a (or an) _____ and a (or an) _____.

2. (10 points) Compute \( \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} \, dS \) where \( \mathbf{F}(x, y, z) = ze^{xy} \mathbf{i} \) and \( S \) is the ellipsoid \( x^2 + 2y^2 + 3z^2 = 1 \).

3. (20 points) The function \( \mathbf{F} = 24xyz \mathbf{i} - 12z^2 \mathbf{k} \) has zero divergence.

(a) Find a vector potential for \( \mathbf{F} \); that is, find \( \mathbf{G} \) whose curl is \( \mathbf{F} \).

(b) Find a vector potential for \( \mathbf{F} \) that has no \( \mathbf{k} \) component.
   (This may or may not be the same as your answer to (a).)

4. (12 points) Let \( D \) be the region in the \( xy \)-plane where \( |x| + |y| \leq 1 \). In other words, it is the region given by
\[
-1 \leq x + y \leq 1 \quad \text{and} \quad -1 \leq x - y \leq 1.
\]
Rewrite \( \iint_D (x - y)^2 e^{x^2 - y^2} \, dx \, dy \) as an integral over \( u \) and \( v \) by using the substitution \( u = x + y, \, v = x - y \). Remember to describe the domain of integration.

Remark: The \( (u, v) \) integral can be evaluated by integrating over \( u \) and then \( v \); however, you are NOT being asked to evaluate it.

END OF EXAM