1. (24 points) Let $F = (x^2 + y^2)i + zj + zk$, and $f = xyz$. In each case, either compute the requested item or briefly explain why it makes no sense.

   (a) the curl of $f$
   (b) the divergence of $f$
   (c) $(F \times \nabla f) \cdot F$
   (d) $f \times F$
   (e) the divergence of $F$
   (f) $(F \times F) \times F$

2. (9 points) For each of the following regions, answer the following questions:

   (i) Is it open?  
   (ii) Is it connected?  
   (iii) Is it simply connected?

You do not need to explain your answers.

   (a) The set of points $(x, y, z)$ with $x^2 + y^2 + z^2 < 2$ and $x^2 + y^2 > 1$.
   (b) The set of points $(x, y, z)$ with $x^2 + y^2 < 2$ and $x^2 + y^2 + z^2 > 1$.
   (c) The set of points $(x, y)$ with $|x| \geq 1$.

3. (5 points) Prove or disprove: The vector field defined by $F = xyzi + xzj + yk$ for all $(x, y, z)$ is conservative.

4. (5 points) Compute $\int_C F \cdot dR$ where $F = xi + xyj$ and $C$ is the straight line segment from the point with position vector $P = (1, 1, 0)$ to the point with position vector $Q = (0, 2, 1)$.

5. (5 points) Let $f(x, y, z) = \tan(\sin(xe^y) + z^2)$ and let $F = \nabla f$. Compute $\int_C F \cdot dR$ where $C$ is the closed curve given by $(x, y, z) = (\cos t, \sin t, e^t \sin t)$, $0 \leq t \leq 2\pi$.

END OF EXAM