YOU MUST SHOW YOUR WORK.

Q1. Evaluate the following integrals. \[ \int_0^8 \sqrt{\frac{2}{t}} \, dt \quad \int e^{x+1} \, dx \]

Q2. (a) Evaluate \( \int \ln t \, dt \).

(b) Evaluate \( \int_1^2 4x(2x - 3)^{50} \, dx \).

(c) Write down an integral for the area of the region enclosed by the three curves

\[ y = e^x, \quad y = x + 1, \quad x = 2. \]

Q3 (a) (8 pts) Evaluate the integrals

\[ \int \frac{\ln x}{x^2} \, dx \quad \int_0^\pi x \cos x \, dx \] (no trig functions in answer).

(b) (4 pts) Set up, but do not evaluate an integral for the volume obtained by rotating the region between \( y = x^4 \) and \( y = 1 \) about the line \( y = -2 \).

Q4 #1 (6 pts) Determine if the following integrals converge or diverge. Remember to give a reason for your answer.

(a) \( \int_{-1}^1 \frac{dx}{x^2} \)

(b) \( \int_1^\infty \frac{dx}{x^2} \)

Q4 #2 (6 pts) Estimating \( \int_{-1}^3 f(x) \, dx \) using the Trapezoidal Rule, I obtained \( T_4 = 8 \) and \( T_8 = 5 \). I also know that \( |f'(x)| \leq 54 \) and \( |f''(x)| \leq 36 \) for \(-1 \leq x \leq 3\).

(a) Find a guaranteed bound on the error in \( T_8 \).

(b) Find a reasonable estimate for the error in \( T_8 \).

Q5. (a) (4 pts) Use Euler’s method with step size \( h = 0.5 \) to estimate \( y(1) \) where \( y'(x) = 2y + 4x \) and \( y(0) = 1 \).

Do the arithmetic!

(b) (4 pts) Find a value of \( A \) so that \( y = x^2 + Ax \) is a solution to the differential equation \( x(dy/dx) - 2y = 3x \).

(c) (4 pts) Set up, but do not evaluate, an integral for the length in the first quadrant of the curve \( x^2 + y^4 = 1 \).

Q6. 1. Express the following as \( a + bi \), where \( a \) and \( b \) are real numbers and do NOT contain trig functions.

(a) \( \frac{10}{2+i} \)

(b) \( e^{(1+i)\pi} \).

2. In each case, indicate if the curve is an ellipse, hyperbola or parabola.

(a) \( x + y = \frac{4}{x-y} \)

(b) \( r = \frac{3}{1-\sin \theta} \).