1. (a) How many four card hands are there where no cards are in the same suit and no cards are the same kind? (“Kind” means A, 2, 3, . . ., 10, J, Q, K.)
   (b) How many 6 card hands contain two 3-of-a-kinds; that is, two sets of cards where each set consists of 3 cards of the same kind?

2. (a) Give an example of a SIMPLE graph with a cut vertex and indicate the cut vertex.
   (b) For the graph in (a), indicate a walk which is NOT a path.
   (c) Give an example of a SIMPLE graph with 3 vertices and 12 edges.

3. An oriented simple graph is a simple graph which has been converted to a digraph by assigning an orientation to each edge.
   (a) Prove that the number of $n$-vertex oriented simple graphs is $3^{n\choose 2}$.
   (b) State and prove a formula for the number of $n$-vertex oriented simple graphs that have exactly $q$ edges.
   Hint: You can construct an oriented simple graph by choosing a simple graph and then orienting each of its edges.

4. The depth of a rooted tree is the number of edges in the longest path from the root to a leaf. A binary rooted tree is a rooted tree in which each vertex either is a leaf or has exactly two children. Some examples are on the blackboard.
   (a) Let $L_n$ be the maximum number of leaves in a binary rooted tree of depth $n$. Prove that
      \[ L_0 = 1 \quad \text{and} \quad L_n = 2L_{n-1} \text{ for } n > 0. \]
   Hint: What happens when the root is removed?
   (b) Using (a), prove that $L_n = 2^n$ for $n \geq 0$.
   (c) Let $l_n$ be the minimum number of leaves in a binary rooted tree of depth $n$. Prove that $l_n = n + 1$. 