There are 125 points total. (So first exam is 20% and this is 25%.)

1. (45 pts.) Indicate whether true or false. Beware of guessing:

   correct answer +5pts.  incorrect answer -3pts.  no answer 0pts

   (a) ___ Every finite set of strings is a CFL.
   (b) ___ The language \{a^n b^n c^n | n > 0\} can be recognized by a (1-stack) PDA.
   (c) ___ A PDA with two stacks can recognize more languages than a standard 1-stack PDA.
   (d) ___ If \(L\) is Turing-decidable, then \(\overline{L}\) is also Turing-decidable.
   (e) ___ A Turing machine with two tapes can recognize more languages than a standard 1-tape Turing machine.
   (f) ___ The language \{a^n b^n c^n d^n | n > 0\} is Turing-recognizable.
   (g) ___ \(L\) is Turing-decidable when \(L\) is the set of strings of digits that represent primes; that is, \(L = \{2, 3, 5, 7, 11, 13, \ldots\}\). \((n\) is a prime if its only positive integer divisors are itself and 1.)
   (h) ___ There exists a Turing machine which can decide if two DFAs are equivalent; that is, whether or not they recognize the same language.
   (i) ___ There exists a Turing machine \(M\) which can decide if a Turing machine will loop on a given input; that is, \(M\)'s input is a description of a machine, say \(T\), and a string, say \(w\), and \(M\) accepts the input if \(T\) does loop on \(w\) and \(M\) rejects the input if \(T\) does not loop on \(w\).

2. (25 pts.) Prove that, if \(L\) and \(M\) are CFLs, then so is \(L \cup M\).
3. (30 pts.) Let $L = \{a^nbc^n \mid n \geq 0\}$.
   
   (a) Construct a context free grammar to generate the language.

   (b) Construct a PDA to recognize the language.

4. (25 pts.) Suppose that both $L$ and $\overline{L}$ are Turing-recognizable. Either (a) prove that $L$ must be Turing-decidable, or (b) give an example of such an $L$ which is not Turing-decidable.