1. (a) This can be done in various ways:

- Best is to take a value on each side of the (50,10) entry and form the finite difference:

  \[ w_s(50,10) \approx \frac{37 - 21}{60 - 40} = \frac{16}{20} = 0.8 \quad \text{and} \quad w_t(50,10) \approx \frac{36 - 19}{15 - 5} = \frac{17}{10}. \]

- Not as good is to take a value at the point and a point on one side (but this is still acceptable):

  \[ w_s(50,10) \approx \frac{37 - 29}{60 - 50} = \frac{8}{10} = 0.8 \quad \text{or} \quad w_s(50,10) \approx \frac{29 - 21}{50 - 40} = \frac{8}{10} \]

  \[ \text{and} \quad w_t(50,10) \approx \frac{36 - 29}{15 - 10} = \frac{7}{5} = 1.4 \quad \text{or} \quad w_t(50,10) \approx \frac{29 - 19}{10 - 5} = \frac{10}{5} = 2. \]

(b) The units of \( w_s \) are feet per knot and those of \( w_t \) feet per hour.

If you have singular or plural where you should not (foot, knots, hours), you will still receive credit.

(c) The answer is \( w(50,10) + w_s(50,10) \times (-1) + w_t(50,15) \times 1 \). You should plug in the numbers you got in (a).

2. By the chain rule

\[ f'(1) = g_x(x(1), y(1))x'(1) + g_y(x(1), y(1))y'(1) \]

\[ = 1 \times 1 + (-2) \times 3 = -5. \]

(b) Since \( f_{xy} = f_{yx} \), we’ll compute \( f_x \) first. It is \( 3x^2y \). Thus \( f_{xy} = \partial(3x^2y)/\partial y = 3x^2 \).

(c) \( |\langle 1, 2 \rangle| = \sqrt{1^2 + 2^2} = \sqrt{5} \). Thus \( u = \langle 1/\sqrt{5}, 2/\sqrt{5} \rangle \). Since \( \nabla f = \langle 2x - 2y, 2x \rangle \), we have \( \nabla f(0,1) = \langle -2, 0 \rangle \) Finally \( D_u f(0,1) = \nabla f(0,1) \cdot u = -2/\sqrt{5} \).

3. The gradient is \( \langle 6x, 2y, 4z \rangle \), which equals \( \langle 6, -8, 4 \rangle \) at \( (1, -4, 1) \). Thus the equation of the plane is

\[ 0 = \langle 6, -8, 4 \rangle \cdot \langle x - 1, y + 4, z - 1 \rangle = 6x - 8y + 4z - 42, \]

which can be rewritten as \( 3x - 4y + 2z = 21 \). Any of these forms, including the dot product form, is acceptable.

4. Since we are given the critical points, we only need to compute \( f_{xx}, f_{xy}, f_{yy} \) and \( D = f_{xx}f_{yy} - (f_{xy})^2 \) there.

| quantity | at \( (x, y) \) | at \( (0,0) \) at \( (1, \sqrt{2}) \) at \( (1, -\sqrt{2}) \) |
|----------|----------------|-------------------|-------------------|-------------------|
| \( f_{xx} \) | -2 | -2 | -2 | -2 |
| \( f_{xy} \) | -2y | 0 | -2\sqrt{2} | 2\sqrt{2} |
| \( f_{yy} \) | 2x - 2 | -2 | 0 | 0 |
| \( D \) | 4(1 - x - y^2) | 4 | -8 | -8 |

By the second derivative test, \( (0,0) \) is a local maximum and the other two are saddle points.