1. (12 points) The table at the bottom of this page gives wave heights \(w\) in feet produced by various wind speeds \(s\) in knots blowing for various lengths of time \(t\) in hours. Thus we have a table of some values of \(w(s,t)\).
   (a) Estimate \(\nabla w(50,15) = \langle w_s(50,15), w_t(50,15) \rangle\).
   (b) What are the units of each of these partial derivatives? (For example — but wrong — knots per hour.)
   (c) Estimate the wave height when a wind of 51 knots has been blowing for 14 hours. You can leave arithmetic like \((27/4) \times 3 - 1\) in your answer. (Of course, this is not the answer.)

2. (12 points) Compute the indicated derivatives.
   (a) \(f'(1)\) given that \(f(t) = g(x(t), y(t))\), \(g(5,0) = -4\), \(g(1,1) = 3\), \(g_x(5,0) = 2\), \(g_y(5,0) = -1\), \(g_x(1,1) = -3\), \(g_y(1,1) = 2\), \(x(1) = 5\), \(x'(1) = 2\), \(y(1) = 0\), and \(y'(1) = 1\).
   (b) \(\frac{\partial f_x(x,y)}{\partial y}\) given that \(f(x,y) = x\sin^2(e^x) + xy^2\).
   (c) \(D_uf(1,0)\) given that \(f(x,y) = x^2 + 2xy\) and \(u\) is a unit vector in the same direction as \((2,1)\).

3. (8 points) Find the equation of the tangent plane to the surface \(2x^2 + 3y^2 + z^2 = 21\) at the point \((1, -1, 4)\).

4. (8 points) Find the local maxima, local minima and saddle points of the function \(f(x,y) = x^2 + y^2 - x^2y + 3\). To help you with your calculations, the critical points are at \((0,0)\), \((\sqrt{2},1)\) and \((-\sqrt{2},1)\).

\[
\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
\text{Duration (hours)} & \text{table of wave} & 5 & 10 & 15 & 20 & 30 & 40 & 50 \\
\hline
\text{height (feet)} & 30 \text{ knots} & 9 & 13 & 16 & 17 & 18 & 19 & 19 \\
\hline
& 40 \text{ knots} & 14 & 21 & 25 & 28 & 31 & 33 & 33 \\
\hline
& 50 \text{ knots} & 19 & 29 & 36 & 40 & 45 & 48 & 50 \\
\hline
& 60 \text{ knots} & 24 & 37 & 47 & 54 & 62 & 67 & 69 \\
\hline
\end{array}
\]